The University of Kansas

Department of Economics

Final **December/16/2019**

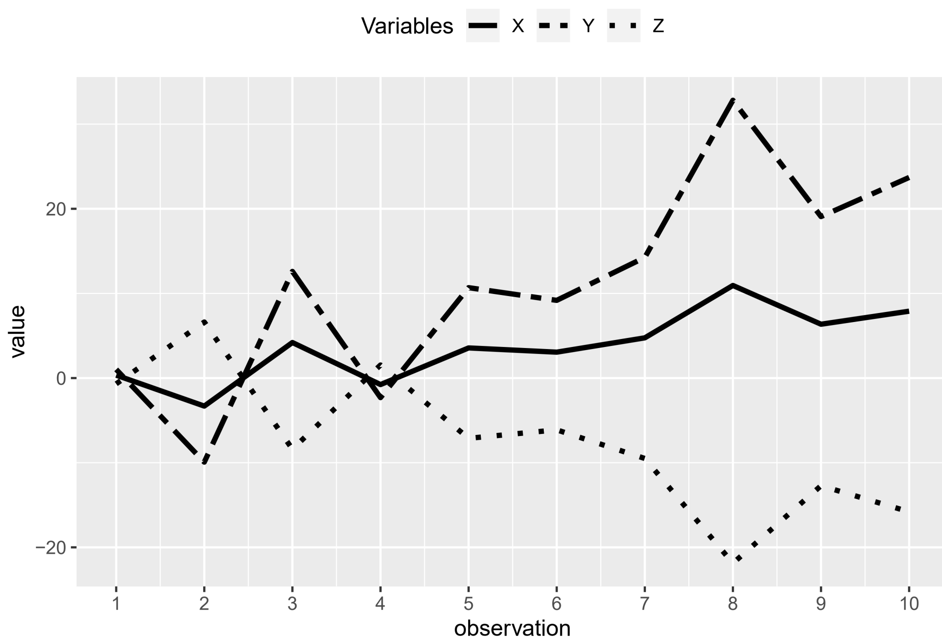
Econ 526 - Introduction to Econometrics **Instructor: Caio Vigo Pereira**

Name:

SECTION A - MULTIPLE CHOICE

3% 1. The figure below shows a sample with 10 observations for three random variables: X, Y and Z. Based on your knowledge of correlation, choose the correct statement below.

A. Based on this sample, $Corr(X, Y) = 1$ and $Corr(X, Z) = 0$
B. Based on this sample, $Corr(X, Z) = 0$ and $Corr(Y, Z) = -1$
C. Based on this sample, $Corr(X, Y) = 1$ and $Corr(X, Z) = -1$
D. Based on this sample, $Corr(Z, X) = 1$ and $Corr(Z, Y) = 1$



3% 2. Consider any multiple linear regression. It is known that under the Gauss-Markov assumptions, the OLS estimators are BLUE. What "B" refers to?

A. That the OLS estimators have the smallest variance among the unbiased estimators.
B. That $E(\hat{\beta}_j^{OLS}) = \beta_j$ for any $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$.
C. That the OLS estimators have the smallest variance among all possible estimators.
D. That the OLS estimators are consistent.

3% 3. Assume that the **Classical Linear Model (CLM)** assumptions hold. What is the distribution of $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)}$?

A. t_{df}, where $df = n - k - 1$
B. $F_{(q,n-k-1)}$
C. $N(0, k^2)$
D. None of the above

3% 4. The ________ is used to compare across models that have different numbers of explanatory variables, but where one is **not** a special case of the other (i.e., **nonnested models**).
A. R^2
B. t *test*
C. *Adjusted* R^2
D. F *test*

3% 5. Consider a multiple linear regression model such as $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + u$. Evaluate the statements below and choose the correct one.

A. Under the Gauss-Markov assumptions, we have $u \sim N(0, 1)$ and $Var(\hat{\beta}_1) = \frac{\sigma^2}{SST_1(1 - R_1^2)}$.

B. The Gauss-Markov assumptions do NOT restrict the distribution of the error term u, but it requires the error term u to be independent of the explanatory variables.

C. The *Classical Linear Model* requires the independent variables to have zero or no collinearity.

D. Under the *Classical Linear Model* assumptions, the error term u is normally distributed with mean 0 and variance σ^2.

3% 6. Which of the following can cause the usual OLS t statistics to be invalid, i.e., not to have t distributions under the null hypothesis?
A. Multicollinearity.
B. Heteroskedasticity.
C. Homoskedasticity.
D. Exogenous variables.

3% 7. Evaluate the statements below and choose the correct one.
A. Whenever we have heteroskedasticity, the OLS estimator still is unbiased and efficient.
B. We have homoskedasticity when the variance of the dependent variable is constant.
C. In order to avoid the issues of identifying heteroskedasticity, we should always use heteroskedasticity-robust standard errors for any sample size when doing inference.
D. None of the above.

SECTION B - TRUE OR FALSE

3% 1. We say that an estimator is unbiased if it has the smallest variance among all other estimators, and we say that an estimator is consistent when it converges in probability to the true parameter.
◯ True ◯ False

3% 2. The following regression model: $log(y) = \beta_0 + \beta_1 x_1 + u$ is also known as constant elasticity model.
◯ True ◯ False

3% 3. Consider the following models:

$$\text{Model 1:} \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

$$\text{Model 2:} \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

Then, $SSR_{model1} \geq SSR_{model2}$ and $SST_{model1} = SST_{model2}$.
◯ True ◯ False

3% 4. Exogenous explanatory variables is not a necessary assumption in order to the OLS estimator to be unbiased, however the assumption $E(u|x_1, \ldots, x_k) = 0$ is necessary.
◯ True ◯ False

3% 5. Multicollinearity violates the Gauss-Markov assumptions, and therefore the OLS estimators are not BLUE.
◯ True ◯ False

3% 6. Given the t statistic, the p-value provides the smallest significance level in order to reject the null hypothesis.
◯ True ◯ False

3% 7. Consider any multiple linear regression. Knowing that you fail to reject H_0 for a specific parameter at 1% significance level, then you must fail to reject the H_0 at 2% significance level, but not necessarily at 0.1% significance level.
◯ True ◯ False

3% 8. Consider the following multiple linear regression model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + u$$

Assume that the 95% confidence interval for β_1 is $[-0.254, 0.878]$.
Therefore, $\hat{\beta}_1$ is statistically different from 1 at 5% significance level.
◯ True ◯ False

SECTION C - SHORT ANSWER

Consider a data set containing a random sample with mortgage loans to individuals in a given year. The data set consists of the following variables (variable's name and description):

loanamt	loan amount for a mortgage in thousands
appinc	applicant income, $1000s
unit	number of units in property
married	=1 if applicant married
atotinc	total monthly income
price	purchase price
liq	liquid assets
cosign	is there a cosigner
netw	net worth
white	=1 if applicant white
mortg	credit history on previous mortgage payments
male	=1 if applicant male

1. (This question refers to **Regression (A)**) Consider the following regression:
[**Notice that the significance level "stars" - *, **, *** - were suppressed in this output**]:

REGRESSION (A)

	Dependent variable: loanamt
atotinc	0.0013
	(0.0002)
liq	0.00003
	(0.00001)
mortg	0.6753
	(1.8140)
price	0.4914
	(0.0100)
Constant	38.6748
	(3.8478)
Observations	1,989
R2	0.7015
Adjusted R2	0.7009
Residual Std. Error	44.0345 (df = 1984)
F Statistic	1,165.8130 (df = 4; 1984)

3% (a) State the null hypothesis that credit history on previous mortgage payments has no *ceteris paribus* effect on the loan amount for a mortgage that an applicant obtains. State the alternative hypothesis that there is an effect (i.e., either positive or negative)? [Two lines answer]

3% (b) Test the hypothesis stated above at the 5% significance level. Find the critical value. [Two lines answer]

3% (c) Do you reject the null hypothesis? Explain the statistical significance of your test at 5% significance level. Would you include *mortg* in a final model explaining the loan amount for a mortgage? Explain. [Three lines answer]

3% (d) Find the 99% confidence interval for β_{price}. Is the variable *price* statistically significant at 1% significance level? [Three lines answer]

2. (This question refers to **Regression (B)**) Consider the following (additional) regression:

REGRESSION (B)

```
===============================================
                        Dependent variable:
                    ---------------------------
                              loanamt
-----------------------------------------------
atotinc                      0.0009***
                             (0.0003)

liq                          0.00003**
                             (0.00001)

mortg                         0.4850
                             (1.8105)

price                        0.4865***
                             (0.0101)

appinc                       0.0498***
                             (0.0139)

unit                         9.8318***
                             (2.2516)

Constant                    26.7082***
                             (4.6017)

-----------------------------------------------
Observations                  1,985
R2                           0.7060
Adjusted R2                  0.7052
Residual Std. Error     43.7525 (df = 1978)
F Statistic         791.8221*** (df = 6; 1978)
===============================================
Note:               *p<0.1; **p<0.05; ***p<0.01
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 2.671e+01  4.602e+00   5.804 7.52e-09 ***
atotinc     9.247e-04  2.616e-04   3.534 0.000418 ***
liq         3.110e-05  1.463e-05   2.127 0.033584 *
mortg       4.850e-01  1.810e+00   0.268 0.788807
price       4.865e-01  1.008e-02  48.260  < 2e-16 ***
appinc      4.983e-02  1.386e-02   3.595 0.000332 ***
unit        9.832e+00  2.252e+00   4.367 1.33e-05 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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3% (a) Which variable(s) is/are statistically significant at 1% significance level. List their names.
[*Hint: No computation required.*] [One line answer]

3% (b) Which variable(s) is/are statistically significant at 10% significance level. List their names.
[*Hint: No computation required.*] [One line answer]

3% (c) Using the data from both regressions, state the null and alternative hypothesis that the applicant's income and number of units in the property are **jointly** statistically significant. [Two lines answer]

3% (d) Write down the unrestricted and the restricted model. [Two lines answer]

3% (e) Test the hypothesis stated above at the 5% significance level. Find the critical value. [Two lines answer]

3% (f) Do you reject the null hypothesis? Explain the statistical significance of your test at 5% significance levels.
[*Hint: Don't forget to use a specific word when explaining the statistical significance.*] [Two lines answer]

REGRESSION (C)

	Dependent variable:
	log(loanamt)
mortg	-0.0756***
	(0.0183)
appinc	0.0019***
	(0.0001)
married	0.1563***
	(0.0215)
Constant	4.7139***
	(0.0387)
Observations	1,986
R2	0.1583
Adjusted R2	0.1570
Residual Std. Error	0.4477 (df = 1982)
F Statistic	124.2273*** (df = 3; 1982)
Note:	*p<0.1; **p<0.05; ***p<0.01

3. (This question refers to **Regression (C)**). Consider the following regression:

3% (a) State the null and alternative hypothesis of the F statistic for overall significance of a regression. Do you reject the null hypothesis? Explain the statistical significance of your test at 1% significance level. [Four lines answer]

3% (b) What is the base group (or benchmark group or reference group) of this regression? What is the estimated intercept for this group? What is the estimated intercept for the other group? Be clear in your answer. Provide the name of the group followed by its estimate. [Three lines answer]

3% (c) What is the **EXACT** estimated average difference in loan amount between applicants who are married or those who are not, controlling for income and credit history on previous mortgage payments? [One line answer]

3% (d) All other factors being equal, is there any statistical evidence that being *married* impacts the loan amount an applicant obtains? Consider three different significance levels: 1%, 5% and 10% (significance level) in your answer. [Three lines answer]

3% (e) Now, suppose you create a new variable called *not_married*, which is defined as $not\_married = 1 - married$. Then you run the following model using the same sample:

$$log(loanamt) = \beta_0 + \beta_1 mortg + \beta_2 appinc + \beta_3 not\_married + u$$

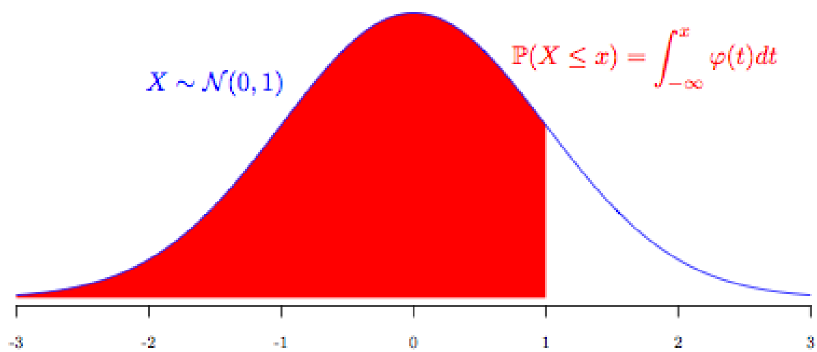
What is the new estimate for β_1? What is the new estimate for β_3? What is the new R^2? Provide a short explanation for your answer. [Four lines answer]

$\boxed{5\%}$ 4. [**Gauss-Markov Theorem**] Under which assumptions does the Gauss-Markov Theorem holds? State and briefly explain each one of them. [One line answer per assumption]

$\boxed{5\%}$ 5. [**Testing Heteroskedasticity**] In order to check if there is heroskedasticity in the data, we need to perform a statistical test. In class you were introduced to two tests: Breusch-Pagan and White tests. Choose one of these tests of heteroskedasticity (either Breusch-Pagan or White test - just one of them), state the null and alternative hypothesis, and the regressions needed to perform this test. Use the following model in your answer:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

Standard Normal Distribution



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Critical Values of the t-distribution

		Significance Level				
1-Tailed:		**.10**	**.05**	**.025**	**.01**	**.005**
2-Tailed:		**.20**	**.10**	**.05**	**.02**	**.01**
	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.499
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
Degrees of Freedom	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	30	1.310	1.697	2.042	2.457	2.750
	40	1.303	1.684	2.021	2.423	2.704
	60	1.296	1.671	2.000	2.390	2.660
	90	1.291	1.662	1.987	2.368	2.632
	120	1.289	1.658	1.980	2.358	2.617
	∞	1.282	1.645	1.960	2.326	2.576

Source: Wooldridge, Jeffrey M. *Introductory Econometrics*, 2015.

1% Critical Values of the F Distribution

	Numerator Degrees of Freedom									
Denominator Degrees of Freedom	1	2	3	4	5	6	7	8	9	10
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Source: Wooldridge, Jeffrey M. *Introductory Econometrics*, 2015.

5% Critical Values of the F Distribution

	Numerator Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	10
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Denominator Degrees of Freedom

Source: Wooldridge, Jeffrey M. *Introductory Econometrics*, 2015.