

Midterm 2
Econ 526 - Introduction to Econometrics

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Name: _____

REGRESSION (A)

Consider a model relating the annual number of crimes on college campuses to the number of police officers and student enrollment. The econometric model is:

$$\log(\text{crime}) = \beta_0 + \beta_1 \text{police} + \beta_2 \log(\text{enroll}) + u$$

where *crime* is total campus crimes, *police* is the number of employed officers and *enroll* is the total enrollment.

The *R* output is:

```

=====
                        Dependent variable:
                        -----
                                log(crime)
                        -----
police                    0.0240***
                           (0.0073)

log(enroll)              0.9767***
                           (0.1373)

Constant                 -4.3758***
                           (1.1990)

=====
Observations              97
R2                       0.6277
Adjusted R2              0.6198
Residual Std. Error      ██████████
F Statistic              79.2389*** ██████████
=====
Note:                    *p<0.1; **p<0.05; ***p<0.01

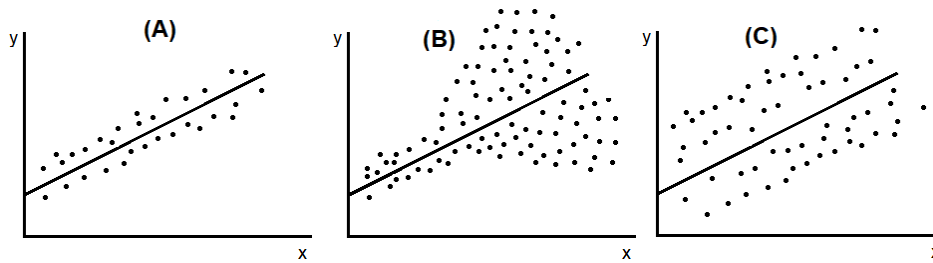
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SECTION A - MULTIPLE CHOICE

- 4% 1. Based on the Regression (A) above, what is the effect on the dependent variable if *police* increases one unit?
- A. $\widehat{\log(\text{crime})}$ will increase 2.40%
 - B. $\widehat{\text{crime}}$ will increase 2.40%
 - C. $\widehat{\log(\text{crime})}$ will increase 0.0240%
 - D. $\widehat{\text{crime}}$ will increase 0.0240%

- 4% 2. Based on the Regression (A) above, what is the effect on the dependent variable if *enroll* increases 10%?
- \widehat{crime} will increase 9.767%
 - $\log(\widehat{crime})$ will increase 0.9767%
 - $\log(\widehat{crime})$ will increase 9.767%
 - \widehat{crime} will increase 0.9767%

Figure 1:



- 4% 3. Consider the models from the Figure 1 above. Which models present heteroskedastic errors?
- (A) and (B)
 - (B) and (C)
 - (A) and (C)
 - Only (B)
- 4% 4. Refer to Figure 1 again. What can you tell about $Var(\hat{\beta})$?
- The $Var(\hat{\beta})$ from model (A) is larger than the $Var(\hat{\beta})$ from model (B), considering all other factors that may affect the variance the same.
 - The $Var(\hat{\beta})$ from model (B) is larger than the $Var(\hat{\beta})$ from model (A), considering all other factors that may affect the variance the same.
 - The $Var(\hat{\beta})$ from model (C) is larger than the $Var(\hat{\beta})$ from model (A), considering all other factors that may affect the variance the same.
 - The $Var(\hat{\beta})$ from model (A) is larger than the $Var(\hat{\beta})$ from model (C), considering all other factors that may affect the variance the same.
- 4% 5. Refer to Figure 1 again. Assuming that assumptions **MLR.1** - **MLR.4** hold, for which models the OLS estimator will be unbiased?
- (A), (B) and (C)
 - (A) and (C) only
 - Only (B)
 - Only (A)

SECTION B - TRUE OR FALSE

- 3% 1. An explanatory variable is said to be exogenous if it is correlated with the error term.
- True False

- 3% 2. (Cobb-Douglas production function) Consider the following model:

$$y_i = \beta_0 x_1^{\beta_1} x_2^{\beta_2} e^u$$

After applying the natural logarithm on both sides, this model is linear in parameters.

True False

- 3% 3. Consider the following models:

$$\text{Model 1: } y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

$$\text{Model 2: } y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

Then, $R_{model1}^2 > R_{model2}^2$.

True False

- 3% 4. Consider the following model:

$$\log(\text{score}) = \beta_0 + \beta_1 \log(\text{hsgpa}) + \beta_2 \log(\text{hsgpa}^3) + u$$

Then, this model suffers from perfect collinearity.

True False

- 3% 5. Consider the following model:

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{exper} + \beta_3 \text{exper}^2 + u$$

Then, this model suffers from perfect collinearity.

True False

- 3% 6. Normally distributed errors is a necessary assumption in order to the OLS estimator to be efficient.

True False

- 3% 7. Whenever $E(u|x_1, \dots, x_k) = 0$, we say that x_1, \dots, x_k are endogenous explanatory variables.

True False

- 3% 8. If you omit an important variable in a multiple linear regression this will cause the OLS estimator to be biased.

True False

- 3% 9. (This question refers to Regression (A) on the first page of your exam.) Based on this model, β_2 represents the elasticity of *crime* with respect to *enroll*.

True False

- 3% 10. (This question refers to Regression (A) on the first page of your exam.) Based on this model, $100 \cdot \beta_1$ represents the semi-elasticity of *crime* with respect to *police*.

True False

SECTION C - SHORT ANSWER

1. (*This question refers to Regression (A) on the first page of your exam.*) Below you can find additional information about this regression:

$$x_1 = \text{police}$$

$$x_2 = \log(\text{crime})$$

$$\sum_{i=1}^{97} (y_i - \hat{y}_i)^2 = 68.18$$

$$\sum_{i=1}^{97} (x_{i1} - \bar{x}_{i1})^2 = 23,454.25$$

- 5% (a) Under the assumption of homoskedastic errors, what is the variance of $\hat{\beta}_{police}$, i.e., what is the formula of $Var(\hat{\beta}_{police})$? [One line answer]
- 5% (b) What is estimator of the variance of u given x_1, x_2 , i.e., the estimator of $Var(u|x_1, x_2)$? [One line answer]
- 5% (c) Based on your answer above, find $\hat{\sigma}^2$. [If you don't have a calculator with you, you may just show your work to get full credit]
- 5% (d) Based on your answer above, find $\hat{\sigma}$, i.e., the Residual Standard Error. [If you don't have a calculator with you, you may just show your work to get full credit]
- 5% (e) Consider the following (additional) regression:

$$\widehat{police} = -93.798 + 12.187 \log(enroll)$$

$$n = 97, R^2 = 0.4206$$

What is the $se(\hat{\beta}_{police})$? Is the $se(\hat{\beta}_{police})$ presented in the regression output table correct? [If you don't have a calculator with you, you may just show your work to get full credit]

2. Gauss-Markov Theorem

- 15% (a) Under which assumptions does the Gauss-Markov theorem holds? State and explain each one of them.
- 5% (b) State precisely the Gauss-Markov theorem? [You may refer to part (a). If you want to use abbreviations or acronyms, you must write what it stands for.]

3. Consider the following regression model:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + u$$

- 5% (a) Specify the least squares function that is minimized by OLS.
- 5% (b) EXTRA POINT Find the OLS first order conditions (FOCs). How many FOCs exist?

EXTRA POINTS

- 2.5% 1. (A-06) The assumption that there are no exact linear relationships among the independent variables in a multiple linear regression model fails if _____, where n is the sample size and k is the number of parameters.
- A. $n > 2$
 - B. $n > k + 1$
 - C. $n < k + 1$
 - D. $n = (k + 1)^2$

- 2.5% 2. (B-11) Consider the following model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

If $\text{corr}(x_1, x_2) = 0.97$, the OLS estimator will provide a biased estimates.

- True False