The University of Kansas

Department of Economics

Final **Dec/14/2018**

Econ 526 - Introduction to Econometrics **Instructor: Caio Vigo Pereira**

Name:

SECTION A - MULTIPLE CHOICE

4% 1. For the past 3 months you verified that **every time** the price of stock A raised, the price of stock B raised, and **every time** the price of stock A dropped, the price of stock B dropped. Then, based on your data, what is the $\text{Corr}(A, B)$? based on Midterm 1, A-3

A. 1
B. -1
C. 0
D. 0.5

4% 2. If $X \sim N(0, \sigma_X^2)$ and $Y \sim N(0, \sigma_Y^2)$. What is the $\text{Cov}(X, Y)$? based on Midterm 1, A-5

A. $E(XY)$
B. $E(XY)^2 - E(X)^2 E(Y)^2$
C. $E(XY)^2 - \mu_X^2 \mu_Y^2$
D. All the above

4% 3. Let X be a random variable and consider a sample with 101 observations. Knowing that $\sum_{i=1}^{n} \frac{X_i}{n} = 0$, and $\sum_{i=1}^{n} X_i^2 = 3256$, what is S^2? based on Quiz 2, A-1

A. $\sqrt{32.56}$
B. 3.256
C. 32.24
D. 32.56

4% 4. Consider the following simple linear regression model: $y = \beta_0 + \beta_1 x + u$. What is the OLS estimator for β_1? based on Midterm 1, A-10

A. $\bar{y} - \hat{\beta}_1 \bar{x}$

B. $\bar{y} - \beta_1 \bar{x}$

C. $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})}$

D. $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

4% 5. Knowing that the estimator of the variance of the error term u given the explanatory variables $x_1, x_2, \ldots, x_k$, i.e., the estimator of $Var(u|x_1, x_2, \ldots, x_k)$ is given by:

$$\hat{\sigma}^2 = \frac{SSR}{df}$$

What is the *Residual Standard Error*: $\hat{\sigma}$? based on Midterm 2, C-1b

A. $\sqrt{\dfrac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-k-1}}$

B. $\sqrt{\dfrac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-k-1}}$

C. $\sqrt{\dfrac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{n-k-1}}$

D. $\sqrt{\dfrac{\sum_{i=1}^{n}(\bar{y} - \hat{y}_i)^2}{n-k}}$

2.5% 6. EXTRA POINTS Among the statements below, which one is **NOT** under the *Classical Linear Model* assumptions?
A. the error term u is normally distributed
B. the error term u is independent of the explanatory variables
C. the error term u has mean 0
D. the variance of the error term u is a function of the explanatory variables

2.5% 7. EXTRA POINTS Which of the following can cause the usual OLS t statistics to be invalid (that is, not to have t distributions under the null hypothesis)?
A. Exogenous variables
B. Multicollinearity
C. Homoskedasticity
D. Endogenous variables

SECTION B - TRUE OR FALSE

3% 1. Let X be a random variable. Among the measures of variability of the distribution of X we have $sd(X)$ and $Var(X)$. based on Quiz 1, B-1
◯ True ◯ False

3% 2. Let $\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$. Then for any random variable X we have that $\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2 = 0$. based on Quiz 1, C-1
◯ True ◯ False

3% 3. Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. random variables with mean μ. The *Law of Large Numbers (LLN)* states that $\bar{Y}$ is an unbiased estimator of μ. based on Quiz 2, B-1
◯ True ◯ False

3% 4. Let $\mathbf{e} = [1, 1, \ldots, 1]$ be a n-dimensional vector of ones, i.e., $\mathbf{e}$ is a constant having the number 1 for each entry (n times). Then $\text{sd}(\mathbf{e}) = \sqrt{1} = 1$. based on Midterm 1, B-2

◯ True ◯ False

3% 5. Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. random variables with mean μ, and variance σ^2. The *Central Limit Theorem (CLT)* states that, for n large, $Z_n = \dfrac{\bar{Y}_n - \mu}{\sigma/\sqrt{n}}$ will converge to a standard Normal distribution **only if** $Y_1, Y_2, \ldots, Y_n$ has Normal distribution. based on Midterm 1, B-3

◯ True ◯ False

3% 6. Depending if we either use the Method of Moments or the Least Squares Method to derive β_0 and β_1 of a simple regression model, we may get different estimators for both parameters. based on Midterm 1, B-9

◯ True ◯ False

3% 7. Consider the following model:

$$log(score) = \beta_0 + \beta_1 log(hsgpa) + \beta_2 \left[log(hsgpa)\right]^3 + u$$

Then, this model suffers from perfect collinearity. based on Midterm 2, B-4

◯ True ◯ False

3% 8. Exogenous explanatory variables is not a necessary assumption in order to the OLS estimator to be unbiased, however the assumption $E(u|x_1, \ldots, x_k) = 0$ is necessary. based on Midterm 2, B-7

◯ True ◯ False

3% 9. The following regression model:

$$log(score) = \beta_0 + \beta_1 log(hsgpa) + u$$

is also known as constant elasticity model, and β_1 is the elasticity of *score* with respect to *hsgpa*. based on Quiz 4, B-2 and B-5

◯ True ◯ False

3% 10. Consider the following models:

$$\text{Model 1:} \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

$$\text{Model 2:} \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

Then, $R^2_{model1} < R^2_{model2}$. based on Midterm 2, B-3

◯ True ◯ False

SECTION C - SHORT ANSWER

Consider a data set containing a random sample with salary information and career statistics for 269 players in the National Basketball Association (NBA). The dataset consists of the following variables (variable's name and description):

wage	annual salary, thousands $
exper	years as professional player
age	age in years
coll	years played in college
games	average games per year
minutes	minutes per season
guard	=1 if guard
forward	=1 if forward
center	=1 if center
points	points per game
rebounds	rebounds per game
assists	assists per game
draft	draft number
allstar	=1 if ever all star
avgmin	minutes per game

1. (This question refers to **Regression (A)** below) Consider the following regression (R output) [**Notice that the significance level "stars" - *, **, *** - were suppressed in this output**]:

REGRESSION (A)

	Dependent variable: log(wage)
exper	0.1289 (0.0354)
age	-0.0585 (0.0350)
coll	-0.0556 (0.0519)
allstar	-0.0038 (0.1376)
avgmin	0.0507 (0.0046)
Constant	6.8871 (0.8442)
Observations	269
R2	0.4913
Adjusted R2	0.4816
Residual Std. Error	0.6346 (df = 263)
F Statistic	50.7930 (df = 5; 263)

5% (a) State the null hypothesis that the number of years played in college has no *ceteris paribus* effect on a NBA's player salary. State the alternative hypothesis that there is an effect.? [Two lines answer]

5% (b) Test the hypothesis stated above at the 1% significance level. Find the critical value. [Two lines answer]

5% (c) Do you reject the null hypothesis? Explain the statistical significance of your test at 1% significance level. [Two lines answer]

2% (d) Would you include *coll* in a final model explaining NBA players salary in terms of years played in college? Why? Explain. [One line answer]

3% (e) A 1 unit increase in *exper* is associate with an increase of ________ in ________ variable? Restate the last sentence filling the gaps. Now, restate it again changing the increase to 10 units.

3% (f) Find the 95% confidence interval for β_{exper}. [One line answer]

2. (This question refers to **Regression (B)** below) Consider the following (additional) regression:

REGRESSION (B)

```
==============================================
                      Dependent variable:
                  ----------------------------
                           log(wage)
----------------------------------------------
exper                      0.1257***
                           (0.0352)

age                         -0.0552
                           (0.0348)

coll                        -0.0370
                           (0.0518)

allstar                     -0.2132
                           (0.1565)

avgmin                     0.0299***
                           (0.0094)

points                     0.0450***
                           (0.0164)

games                       -0.0001
                           (0.0025)

Constant                   6.8158***
                           (0.8361)

----------------------------------------------
Observations                  269
R2                           0.5056
Adjusted R2                  0.4923
Residual Std. Error     0.6280 (df = 261)
F Statistic         38.1252*** (df = 7; 261)
==============================================
Note:              *p<0.1; **p<0.05; ***p<0.01
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.8158193  0.8360829   8.152  1.5e-14 ***
exper        0.1256580  0.0351871   3.571 0.000423 ***
age         -0.0552317  0.0347665  -1.589 0.113350
coll        -0.0369725  0.0518374  -0.713 0.476335
allstar     -0.2131675  0.1565311  -1.362 0.174428
avgmin       0.0299440  0.0093665   3.197 0.001560 **
points       0.0449557  0.0163604   2.748 0.006417 **
games       -0.0001336  0.0025255  -0.053 0.957843
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

2% (a) Which variables are statistically significant at 1% significance level. List their names [*Hint:* no computation required.] [One line answer]

4% (b) Using the data from both regressions, state the null and alternative hypothesis that *points* and *games* are **jointly** significant. Write down the unrestricted and the restricted model. [Four lines answer]

4% (c) Test the hypothesis stated above at the 1% significance level. Find the critical value. Test the same hypothesis again at the 5% significance level. Find the critical value. [Four lines answer]

4% (d) Do you reject the null hypothesis? Explain the statistical significance of your test at 1% and 5% significance levels. [*Hint:* Don't forget to use a specific word when explaining the statistical significance.] [Four lines answer]

3. (This question refers to **Regression (C)** below). Answer the questions below knowing that in this data set we classify a basketball player in one of the following three categories: guard, forward and center.

REGRESSION (C)

```
===============================================
                      Dependent variable:
                  ---------------------------
                             wage
-----------------------------------------------
exper                      224.4807***
                            (48.2237)

age                        -110.9587**
                            (48.3493)

forward                     108.5672
                           (112.4999)

Constant                  3,269.9330***
                          (1,096.8610)

-----------------------------------------------
Observations                   269
R2                            0.1862
Adjusted R2                   0.1770
Residual Std. Error     906.9850 (df = 265)
F Statistic          20.2135*** (df = 3; 265)
===============================================
Note:               *p<0.1; **p<0.05; ***p<0.01
```

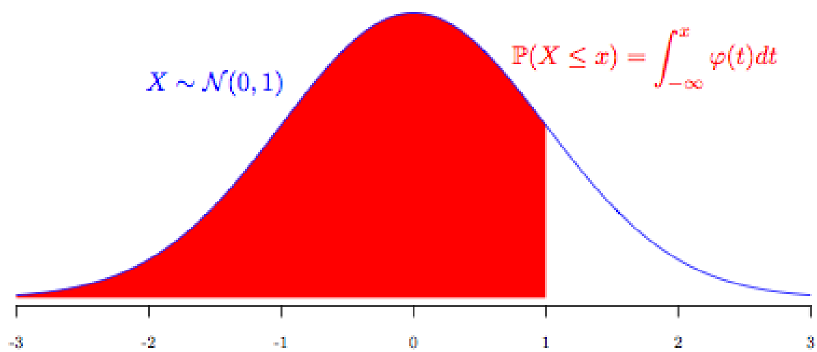
3% (a) State the null and alternative hypothesis of the F statistic for overall significance of a regression. Do you reject the null hypothesis? Explain the statistical significance of your test at 1% significance level. [Three lines answer] [Three lines answer]

2% (b) What is the estimated average difference in salary between being a forward or not, for players with the same *exper* and *age*? [*Hint: Use the correct measure unit*] [One line answer]

5% (c) All other factors being equal, is there any statistical evidence that being a *forward* player impacts the annual salary of a NBA player? Consider three different significance levels: 1%, 5% and 10% (significance level) in your answer. [Two lines answer] [Three lines answer]

3% (d) Suppose we create a variable *not_foward*, such that *not_foward* = *guard* + *center*. Can we add the variable *not_foward* to the above **Regression (C)**? Yes or no. Why? [Three lines answer]

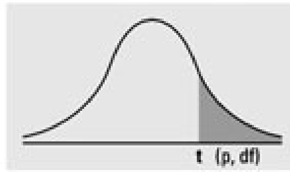
Standard Normal Distribution



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

t-distribution

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

1% Critical Values of the F Distribution

	Numerator Degrees of Freedom									
Denominator Degrees of Freedom	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

5% Critical Values of the F Distribution

Denominator Degrees of Freedom	Numerator Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	10
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83